ME265: Thermal Engineering & Heat Transfer

Chapters
1. Energy Scenario
2. Thermodynamics
3. Mechanical Devices & Systems
4. Heat Transfer

4.1 Introduction	
4.2 Conduction	4.2.1 Conduction Equations 4.2.2 Boundary & Initial conditions 4.2.3 Steady Heat Conduction 4.2.4 Transient Heat Conduction
4.3 Convection	
4.4 Radiation	
4.5 Heat Exchanger	

4.2.3	**Steady Heat Conduction**	
	4.2.3.1	**Solutions to 1D-SS HC problems**
	4.2.3.2	**Thermal resistances**
	4.2.3.3	**R-values of Insulation**
	4.2.3.4	**Critical thickness of insulation**
	4.2.3.5	**Thermal Analysis of fins**

4.2.3 Steady Heat Conduction

For one dimensional (1D) homogeneous, isotropic solids **without heat generation**:

Coordinate System	**Governing Equation (Transient)**	**Governing Equation (Steady State)**
Cartesian	$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{d^2 T}{dx^2} = 0$
Cylindrical	$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$
Spherical	$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$	$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$

k → Thermal conductivity, W/m.K
α → Thermal diffusivity = k/ρc, m^2/s

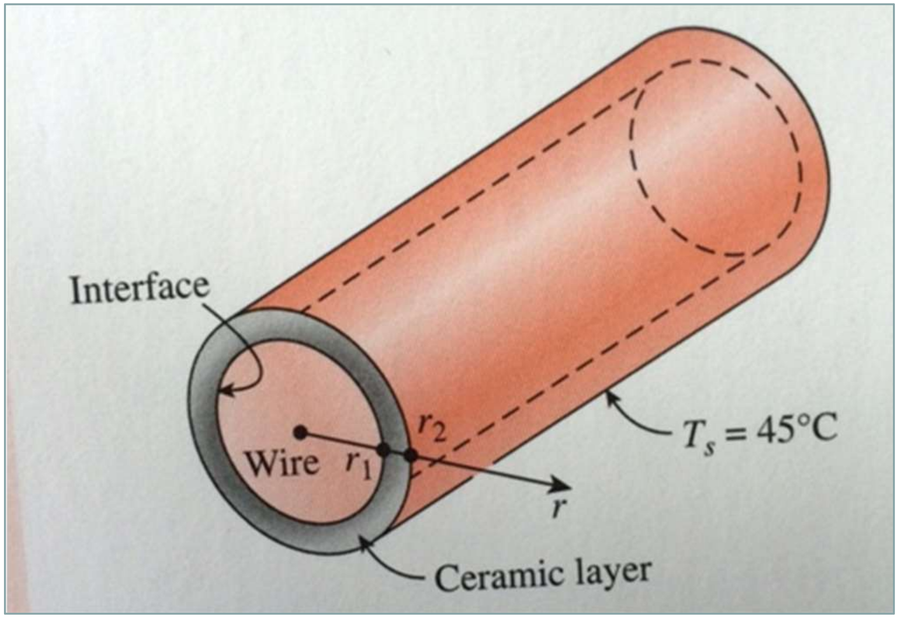
4.2.3 Steady Heat Conduction

- Boundary conditions:
  - Thermal conditions at the boundaries of the medium
  - Types
    1. Temperature BC—1st kind
    2. Heat flux BC—2nd kind
    3. Convection BC—3rd kind
    4. Interface BC

4.2.3 Steady Heat Conduction

EP#2.2 Temperature distribution in a resistance heater embedded in ceramic layer (Cengel et al Example 2-19)

Consider a long resistance wire of radius 0.2 cm and thermal conductivity 15 W/m.K in which heat is generated uniformly as a result of resistance heating at a constant rate of 50 W/cm^3. The wire is embedded in a 0.5-cm-thick layer of ceramic (k=1.2 W/m.K). If the outer surface temperature of the ceramic layer is measured to be 45^{o}C, determine the temperatures at the center of the wire and the interface of the wire and ceramic layer under steady state conditions.



EP#2.2 (Soln)

EP#2.2 (Soln)

4.2.3 Steady Heat Conduction

For one dimensional (1D) medium with constant properties and with no heat generation:

C/System	Governing Equation	Thermal Condition	Examples
Cartesian	$\frac{d^2T}{dx^2} = 0$	Steady State	Slab, Plane wall
Cylindrical	$\frac{d}{dr}\left(r\frac{dT}{dr}\right) = 0$	Steady State	Solid & hollow pipe, tube, wire, etc.
Spherical	$\frac{d}{dr}\left(r^2\frac{dT}{dr}\right) = 0$	Steady State	Solid & hollow sphere

These governing equations will be used to illustrate **Thermal resistances**

4.2.3 Steady Heat Conduction

4.2.3.2 Thermal Resistances, R

$$Q = \frac{T_1 - T_2}{R}$$

R is the thermal resistance and is associated with 1st kind BC

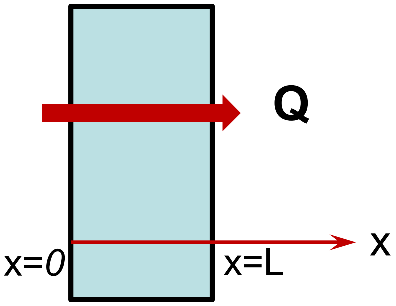
R depends on

- modes of heat transfer
- geometry
- properties of medium

4.2.3 Steady Heat Conduction

4.2.3.2 Thermal Resistances: for conduction, R_{cond}

1D-SS heat conduction in solids with 1st kind (temperature) BC



Plane wall

Math model

$$\frac{d^2T}{dx^2} = 0 \text{ … … for } 0<x<L$$
$$T = T_1 \text{ … … at } x=0$$
$$T = T_2 \text{ … … at } x=L$$

Solving the math model, we get: $\frac{dT}{dx} = C_1$ and $T = C_1 x + C_2$

$$C_2 = T_2 \; and \; C_1 = (T_2 - T_1)/L$$

$$Q = -kA\frac{dT}{dx} = -kAC_1 = \frac{T_1 - T_2}{\frac{L}{kA}} = \frac{T_1 - T_2}{R} \Rightarrow \quad R = \frac{L}{kA}$$